THE APPROXIMATION OF SOLUTION FOR UNFORCED DUFFING OSCILLATOR USING LAPLACE ADOMAIN DECOMPOSITION METHOD (LADM)

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ABSTRACT: *In this paper, we will solve an unforced Duffing equation with its initial value. Which is an example of nonlinear oscillator differential equations and find the approximation solution of this equation using Laplace Adomain Decomposition Method (LADM).*

Key words: Approximation solution, Nonlinear oscillator, Duffing equation, Laplace Adomain Decomposition Method (LADM), initial value.

1- INTRODUCTION:

In 1980 George Adomain introduced a powerful method to solve nonlinear functional equation. Since then, this method is known as the Adomain decomposition method (ADM). The technique is based on a decomposition of solution of nonlinear equation in a series of function[1-6] .the series obtained from a polynomial generated by power series expansion of analytica function .

2- Modified Adomain Decomposition Method (ADM):

The principle of the Adomian decomposition method (ADM) when applied to general nonlinear equation is in the following form[7-9]:

$$Lu + Ru + Nu = g \qquad (1)$$

The linear terms decomposition $(Lu + Ru)$, L is an easily invertible linear operator , R is the remaining linear part . The nonlinear terms are represented by Nu .

Take Laplace inverse to both side in equation (1)

$$u = \mathcal{L}^{-1}\{g\} - \mathcal{L}^{-1}\{Ru\} - \mathcal{L}^{-1}\{Nu\} \qquad (2)$$

The decomposition method represents the solution of equation (2) as the following infinite series :

$$u = \sum_{n=0}^{\infty} u_n = u_0 + u_1 + u_2 + \cdots + u_n \qquad (3)$$

$$u_0 = \mathcal{L}^{-1}\{g\} \qquad (4)$$

$$u = u_0 - \mathcal{L}^{-1}\{Ru\} - \mathcal{L}^{-1}\{Nu\} \qquad (5)$$

The nonlinear operator $Nu = F(u)$ in to a particular series of polynomial .

$$Nu = \sum_{n=0}^{\infty} A_n \qquad (6)$$

Where A_n , are adomian polynomials , which are defined :

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F \sum_{i=0}^{\infty} \lambda^i u_i\right]\Big|_{\lambda=0} \qquad (7)$$

$$A_0 = F \qquad A_1 = \frac{d}{d\lambda} F(u_0 + u_1\lambda)|_{\lambda=0} = u_1\grave{F}(u_0)$$

$$A_2 = u_2\grave{F}(u_0) + \frac{1}{2!}u_1^2 F''(u_0)$$

$$A_3 = u_3 F^{'}(u_0) + u_1 u_2 F^{''}(u_0) + \frac{1}{3!}u_1^3 F^{(3)}(u_0)$$

$$A_4 = u_4 F^{'}(u_0) + \left(u_1 u_3 + \frac{1}{2!}u_1^2\right) F^{''}(u_0) + \frac{1}{2!}u_1^2 u_2 F^{(3)}(u_0) + \frac{1}{4!}u_1^4 F^{(4)}(u_0)$$

$$A_5 = u_5 F^{'}(u_0) + (u_2 u_3 + u_1 u_4) F^{''}(u_0) + \frac{1}{2!}(u_1^2 u_3 + u_2^2 u_1) F^{(3)}(u_0) + \frac{1}{3!}u_1^3 u_2 u_1^4 F^{(4)}(u_0) + \frac{1}{5!}F^{(5)}(u_0) \qquad (8)$$

Equation (1) becomes :

$$\sum_{n=0}^{\infty} u_n = \mathcal{L}^{-1}\{g\} - \mathcal{L}^{-1}R\sum u_n - \mathcal{L}^{-1}\sum A_n \qquad (9)$$

$$u_0 = \mathcal{L}^{-1}\{g\} + \phi \quad , \quad u_1 = -\mathcal{L}^{-1}Ru_0 - \mathcal{L}^{-1}A_0$$
$$u_2 = -\mathcal{L}^{-1}Ru_1 - \mathcal{L}^{-1}A_1 \quad , u_3 = -\mathcal{L}^{-1}Ru_2 - \mathcal{L}^{-1}A_2$$
$$u_n = -\mathcal{L}^{-1}Ru_{n-1} - \mathcal{L}^{-1}A_{n-1} \qquad (10)$$

Where ϕ is the initial condition ,

The approximation of the solution :

$$U_M = \sum_{n=0}^{M} u_n \qquad (11)$$

with the $\lim_{M\to\infty} U_M = u \qquad (12)$

3- Appling Laplace Adomain Decomposition Method (LADM) for Duffing equation:

Duffing equation:

$$\ddot{x} + \delta\dot{x} + \beta x + \alpha x^3 = f\cos(\omega t) \qquad (13)$$

When $\delta = \beta = \alpha = 1$, and $f = 0$ the above equation become

$$\ddot{x} + \dot{x} + x + x^3 = 0 \qquad (14)$$

Let we apply the boundary conditions

$$x(0) = 1 \; and \; \dot{x}(0) = 0 \qquad (15)$$

Let us apply the technique Laplace Adomain Decomposition Method, which described in pervious chapter, to solve Duffing equation. First apply Laplace transform in eq.(14) we get

$$L\{\ddot{x}(t)\} + L\{\dot{x}(t)\} + L\{x(t)\} + L\{x^3(t)\} = 0$$

From the properties of the Laplace transform of the derivative

$$L\{\ddot{x}(t)\} = s^2X(s) - sx(0) - \dot{x}(0)$$
$$L\{\dot{x}(t)\} = sX(s) - x(0)$$
$$L\{x(t)\} = X(s)$$
$$s^2X(s) - sx(0) - \dot{x}(0) + sX(s) - x(0) + X(s) + L\{x^3(t)\} = 0$$

Applying boundary conditions

$$X(s)(s^2 + s + 1) = (s + 1) - L\{x^3(t)\}$$

$$X(s) = \frac{s+1}{(s^2+s+1)} - \frac{1}{(s^2+s+1)}L\{x^3(t)\} \qquad (16)$$

$L\{x^3(t)\}$, *nonlinear term*

Applying inverse (L.T)

$$x(t) = L^{-1}\{X(s)\} = L^{-1}\left[\frac{s+1}{(s^2+s+1)}\right] - L^{-1}\left[\frac{1}{(s^2+s+1)}L\{x^3(t)\}\right] \qquad (17)$$

Now replace the linear terms $x(t)$ by an infinite series.

$$x(t) = \sum_{n=0}^{\infty} x_n(t) \quad (18)$$

The nonlinear operator x^3, is defined by an infinite series.

$$x^3 = \sum_{n=0}^{\infty} A_n\,(x_0, x_1, \dots \dots) \quad (19)$$

A_n called Adomian polynomials of $(x_0, x_1, \dots \dots, x_n)$ can be calculated by .

$$A_n(x_0, x_1, \dots, x_n) = \frac{1}{n!}\left(\frac{d^n}{d\lambda^n}\right)\left[N\left(\sum_{i=0}^{\infty} \lambda^i x_i\right)\right] \quad , \quad n = 0,1,2,3,..$$

The first six Adomain polynomial of A_n as

$$A_0 = x_0^3 \quad , \quad A_1 = 3x_0^2 x_1$$

$$A_2 = 3x_0^2 x_2 + 3x_0 x_1^2$$

$$A_3 = 3x_0^2 x_2 + 6x_0 x_1 x_2 + x_1^3$$

$$A_4 = 3x_0^2 x_4 + 6x_0 x_1 x_3 + 3x_0 x_2^2 + 3x_1^2 x_2$$

$$A_5 = 3x_1^2 x_3 + 3x_1 x_2^2 + 6x_0 x_2 x_3 + 6x_0 x_1 x_4 + 3x_0^2 x_5$$

(20)

And so on

Now the equation (18) become.

$$\sum_{n=0}^{\infty} x_n(t) = L^{-1}\left[\frac{s+1}{(s^2+s+1)}\right] - L^{-1}\left[\frac{1}{(s^2+s+1)} L\{\sum_{n=0}^{\infty} A_n\}\right] \quad (21)$$

$$x_0(t) = L^{-1}\left[\frac{s+1}{(s^2+s+1)}\right] \quad (22)$$

$$x_1(t) = L^{-1}\left[\frac{1}{(s^2+s+1)} L\{A_0\}\right] \quad (23)$$

$$x_2(t) = L^{-1}\left[\frac{1}{(s^2+s+1)} L\{A_1\}\right] \quad (24)$$

$$x_3(t) = L^{-1}\left[\frac{1}{(s^2+s+1)} L\{A_2\}\right] \quad (25)$$

$$x_n(t) = L^{-1}\left[\frac{1}{(s^2+s+1)} L\{A_{n-1}\}\right] \quad (26)$$

Now

$$x_0(t) = L^{-1}\left[\frac{s+1}{(s^2+s+1)}\right]$$

$$x_0(t) = \frac{e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2} t\right)}{\sqrt{3}} + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2} t\right) \quad (27)$$

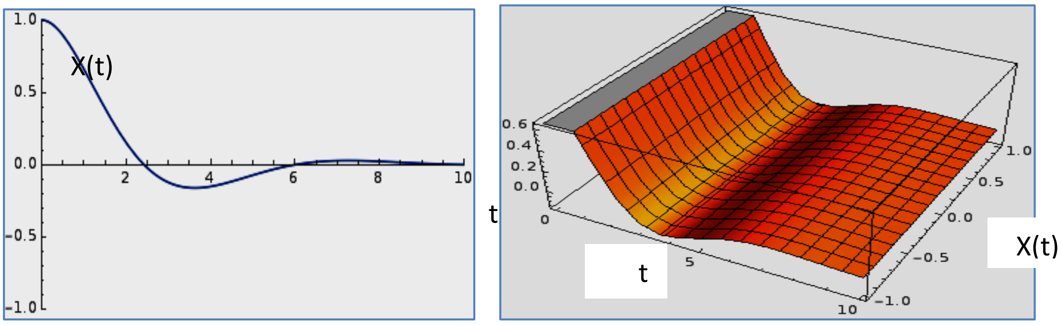

Fig (1): the initial approximation solution of the Duffing equation.

Notice present the damped of displacement x of a spring with time t, the system returns to equilibrium quickly without oscillating. For example Pendulum suspended in water.

Substituting eq.(27)in eq.(21) we get

$$\sum_{n=0}^{\infty} x_n(t) = \frac{e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2} t\right)}{\sqrt{3}} + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2} t\right) - L^{-1}\left[\frac{1}{(s^2+s+1)} L\{\sum_{n=0}^{\infty} A_n\}\right] \quad (28)$$

Generally, the term x_{n+1} , is given by

$$x_{n+1}(t) = -L^{-1}\left[\frac{1}{(s^2+s+1)} L\{A_n\}\right] \quad (29)$$

Substituting $n = 0$, in eq.(29) we get

$$x_1(t) = -L^{-1}\left[\frac{1}{(s^2+s+1)} L\{A_0\}\right] \quad (30)$$

From eq.(22) we get

$$A_0 = x_0^3$$

$$A_0 = \left(\frac{e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2} t\right)}{\sqrt{3}} + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2} t\right)\right)^3 \quad (31)$$

Substituting A_0 in eq.(30) we get

$$x_1(t) = -L^{-1}\left[\frac{1}{(s^2+s+1)} L\left\{\left(\frac{e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2} t\right)}{\sqrt{3}} + e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2} t\right)\right)^3\right\}\right] \quad (32)$$

$$x_1(t) == -L^{-1}\left[\frac{1}{(s^2+s+1)}\left(\frac{s^3+6s^2+18s+21}{(s^2+3s+3)(s^2+3s+9)}\right)\right] \quad (33)$$

$$x_1(t) = \frac{1}{78\sqrt{3}} e^{-3t/2}\left[-138e^t \sin\left(\frac{\sqrt{3}}{2} t\right) + 39 \sin\left(\frac{\sqrt{3}}{2} t\right) + 5 \sin\left(\frac{3\sqrt{3}}{2} t\right) + 42\sqrt{3}e^t \cos\left(\frac{\sqrt{3}}{2} t\right) - 39\sqrt{3} \cos\left(\frac{\sqrt{3}}{2} t\right) - 3\sqrt{3} \cos\left(\frac{3\sqrt{3}}{2} t\right)\right] \quad (34)$$

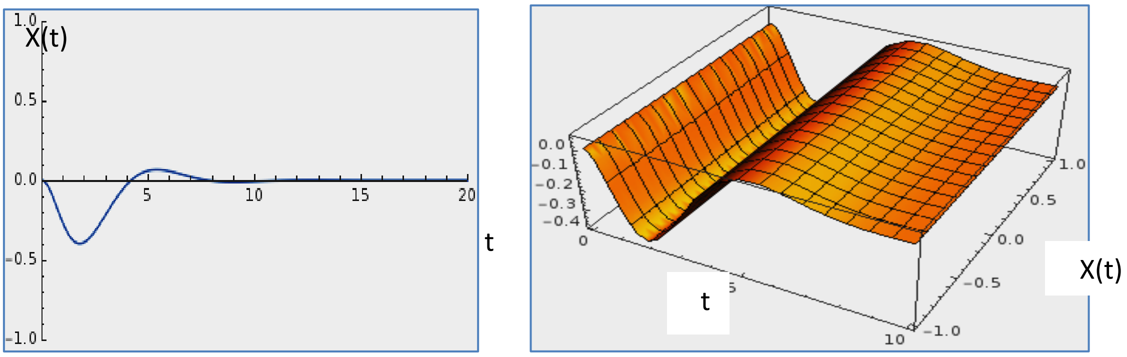

Fig (2): the first approximation solution of the Duffing equation.

Notice present the damped of displacement x of a spring with time t, the system returns to equilibrium quickly without oscillating.

From eq.(20) to find A_1

$A_1 = 3x_0^2 x_1$

$A_1 =$

$$3\left(\frac{e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}} + e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\right)^2 \left(\frac{1}{78\sqrt{3}}e^{-3t/2}\left[-138e^t\sin\left(\frac{\sqrt{3}}{2}t\right) + 39\sin\left(\frac{\sqrt{3}}{2}t\right) + 5\sin\left(\frac{3\sqrt{3}}{2}t\right) + 42\sqrt{3}e^t\cos\left(\frac{\sqrt{3}}{2}t\right) - 39\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}t\right) - 3\sqrt{3}\cos\left(\frac{3\sqrt{3}}{2}t\right)\right]\right) \quad (35)$$

$$x_2(t) = -L^{-1}\left[\frac{1}{(s^2+s+1)}L\{A_1\}\right] \quad (36)$$

Substituting eq.(35) in eq.(36) we get

$$x_2(t) = -L^{-1}\left[\frac{1}{(s^2+s+1)}L\left\{3\left(\frac{e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}} + e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\right)^2 \left(\frac{1}{78\sqrt{3}}e^{-3t/2}\left[-138e^t\sin\left(\frac{\sqrt{3}}{2}t\right) + 39\sin\left(\frac{\sqrt{3}}{2}t\right) + 5\sin\left(\frac{3\sqrt{3}}{2}t\right) + 42\sqrt{3}e^t\cos\left(\frac{\sqrt{3}}{2}t\right) - 39\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}t\right) - 3\sqrt{3}\cos\left(\frac{3\sqrt{3}}{2}t\right)\right]\right)\right\}\right]$$

$$x_2(t) = -L^{-1}\left[\frac{1}{s^2+s+1}L\left[\left[\frac{e^{-\frac{5t}{2}}\sin(\sqrt{3}t)+1}{26\sqrt{3}}\right]\left[-138e^t\sin\left(\frac{\sqrt{3}}{2}t\right) + 39\sin\left(\frac{\sqrt{3}}{2}t\right) + 5\sin\left(\frac{3\sqrt{3}}{2}t\right) + 42\sqrt{3}e^t\cos\left(\frac{\sqrt{3}}{2}t\right) - 39\sqrt{3}\cos\left(\frac{\sqrt{3}}{2}t\right) - 3\sqrt{3}\cos\left(\frac{3\sqrt{3}}{2}t\right)\right]\right]\right]$$

$$x_2(t) = -L^{-1}\left[\frac{1}{s^2+s+1}\left[\frac{21\left(s+\frac{3}{2}\right)}{13\left(\left(s+\frac{3}{2}\right)^2+\frac{3}{4}\right)} - \frac{69\sqrt{3}\left(s+\frac{3}{2}\right)}{13\left(\left(s+\frac{3}{2}\right)^2+\frac{3}{4}\right)\left(\left(s+\frac{3}{2}\right)^2+\frac{27}{4}\right)} - \frac{69}{26\left(\left(s+\frac{3}{2}\right)^2+\frac{3}{4}\right)} + \frac{21\sqrt{3}\left(\left(s+\frac{3}{2}\right)^2+\frac{9}{4}\right)^2}{13\left(\left(s+\frac{3}{2}\right)^2+\frac{3}{4}\right)\left(\left(s+\frac{3}{2}\right)^2+\frac{27}{4}\right)} - \frac{3\left(s+\frac{5}{2}\right)}{2\left(\left(s+\frac{5}{2}\right)^2+\frac{3}{4}\right)} + \frac{3}{4\left(\left(s+\frac{5}{2}\right)^2+\frac{3}{4}\right)} - \frac{3\left(s+\frac{5}{2}\right)}{26\left(\left(s+\frac{5}{2}\right)^2+\frac{27}{4}\right)} - \frac{3\sqrt{3}\left(\left(s+\frac{5}{2}\right)^2+\frac{9}{4}\right)}{2\left(\left(s+\frac{5}{2}\right)^2+\frac{3}{4}\right)\left(\left(s+\frac{5}{2}\right)^2+\frac{27}{4}\right)} + \frac{3\sqrt{3}\left(s+\frac{5}{2}\right)}{26\left(\left(s+\frac{5}{2}\right)^2+\frac{3}{2}\right)\left(\left(s+\frac{5}{2}\right)^2+\frac{75}{4}\right)} + \frac{15}{52\left(\left(s+\frac{5}{2}\right)^2+\frac{27}{4}\right)} + \frac{15\sqrt{3}\left(s+\frac{5}{2}\right)}{26\left(\left(s+\frac{5}{2}\right)^2+\frac{3}{4}\right)\left(\left(s+\frac{5}{2}\right)^2+\frac{75}{4}\right)} - \frac{3\sqrt{3}\left(\left(s+\frac{5}{2}\right)^2-\frac{15}{4}\right)}{26\left(\left(s+\frac{5}{2}\right)^2+\frac{3}{4}\right)\left(\left(s+\frac{5}{2}\right)^2+\frac{75}{4}\right)}\right]\right] \quad (37)$$

$$
\begin{aligned}
x_2(t) = & -\frac{35\sqrt{3}}{52}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{21\sqrt{3}}{52}e^{-\frac{3t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{21}{52}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{21}{52}e^{-\frac{3t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&+\frac{138}{169}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-\frac{69}{104}e^{-\frac{3t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{207}{1352}e^{-\frac{3t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)-\frac{207\sqrt{3}}{676}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&+\frac{23\sqrt{3}}{104}e^{-\frac{3t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{115\sqrt{3}}{1352}e^{-\frac{3t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right)+\frac{23}{52}\sqrt{3}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{23}{52}\sqrt{3}e^{-\frac{3t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)\\
&-\frac{69}{52}\sqrt{3}\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{69}{52}e^{-\frac{3t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{2793}{1352}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-\frac{63}{208}e^{-\frac{3t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)\\
&-\frac{945}{2704}e^{-\frac{3t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)+\frac{63}{676}\sqrt{3}\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{63}{208}\sqrt{3}\,e^{-\frac{3t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&+\frac{567}{2704}\sqrt{3}\,e^{-\frac{3t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right)-\frac{11}{28}\sqrt{3}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{3}{28}\sqrt{3}e^{-\frac{5t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{3}{14}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&-\frac{3}{14}e^{-\frac{5t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-\frac{\sqrt{3}}{14}e^{-\frac{5t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{3}{28}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{3}{28}e^{-\frac{5t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&+\frac{23}{1456}\sqrt{3}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-\frac{9}{1456}\sqrt{3}e^{-\frac{5t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)+\frac{3}{1456}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{3}{1456}e^{-\frac{5t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right)\\
&+\frac{69}{224}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{24}{224}e^{-\frac{5t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-\frac{3}{224}e^{-\frac{5t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)-\frac{3}{32}\sqrt{3}\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&+\frac{3}{56}\sqrt{3}\,e^{-\frac{5t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{9}{224}\sqrt{3}\,e^{-\frac{5t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right)+\frac{1284}{2437}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)\\
&-\frac{108}{2437}\sqrt{21}e^{-\frac{5t}{2}}\sin\left(\frac{9}{2\sqrt{7}}t\right)-\frac{78}{2437}\sqrt{3}\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)+\frac{78}{2437}e^{-\frac{5t}{2}}\cos\left(\frac{9}{2\sqrt{7}}t\right)\\
&-\frac{75}{1456}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{5}{1456}e^{-\frac{5t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)+\frac{15}{1456}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{15}{1456}e^{-\frac{5t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right)\\
&-\frac{1725}{90272}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+\frac{5}{728}e^{-\frac{5t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-\frac{25}{12896}e^{-\frac{5t}{2}}\sin\left(\frac{5\sqrt{3}}{2}t\right)+\frac{495}{90272}\sqrt{3}\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&-\frac{5}{364\sqrt{3}}e^{-\frac{5t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{35}{12896\sqrt{3}}e^{-\frac{5t}{2}}\cos\left(\frac{5\sqrt{3}}{2}t\right)+\frac{411}{90272}e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-\frac{3}{364}e^{-\frac{5t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)\\
&-\frac{21}{12896}e^{-\frac{5t}{2}}\sin\left(\frac{5\sqrt{3}}{2}t\right)+\frac{267}{90272}\sqrt{3}\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{3}{728}\sqrt{3}\,e^{-\frac{5t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)\\
&+\frac{15}{12896}\sqrt{3}\,e^{-\frac{5t}{2}}\cos\left(\frac{5\sqrt{3}}{2}t\right)
\end{aligned}
$$

$$
\begin{aligned}
x_2(t) = &\\
&-1.7e^{-\frac{t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+0.5e^{-\frac{3t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)+0.17e^{-\frac{5t}{2}}\sin\left(\frac{\sqrt{3}}{2}t\right)-2.16\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)+0.78\,e^{-\frac{3t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)-\\
&0.2e^{-\frac{3t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)+0.5\,e^{-\frac{3t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right)-0.26\,e^{-\frac{5t}{2}}\cos\left(\frac{\sqrt{3}}{2}t\right)+0.009e^{-\frac{5t}{2}}\sin\left(\frac{3\sqrt{3}}{2}t\right)+0.06\,e^{-\frac{5t}{2}}\cos\left(\frac{3\sqrt{3}}{2}t\right)-\\
&0.2e^{-\frac{5t}{2}}\sin\left(\frac{9}{2\sqrt{7}}t\right)+0.06e^{-\frac{5t}{2}}\cos\left(\frac{9}{2\sqrt{7}}t\right)-0.004e^{-\frac{5t}{2}}\sin\left(\frac{5\sqrt{3}}{2}t\right)-0.003\,e^{-\frac{5t}{2}}\cos\left(\frac{5\sqrt{3}}{2}t\right) \quad .\quad .(38)
\end{aligned}
$$

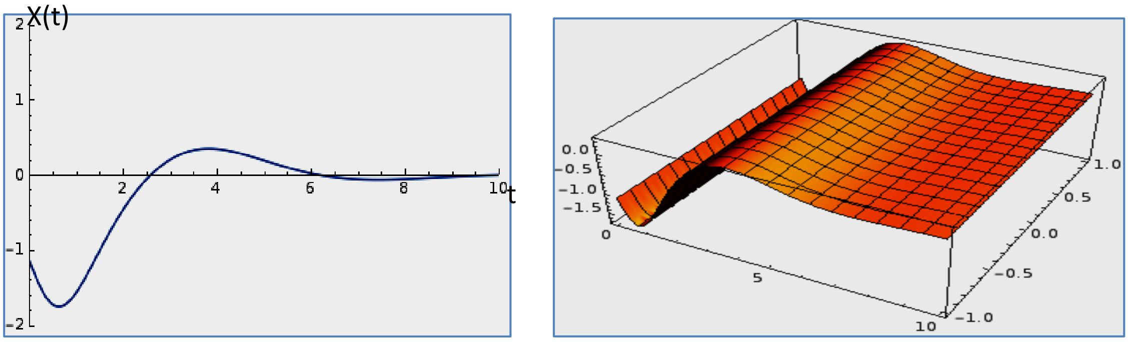

Fig (3): the second approximation solution of the Duffing equation.

Notice present the damped of displacement x of a spring with time t, the system returns to equilibrium quickly without oscillating .

The approximation solution,

$$x(t) = x_0 + x_1 + x_2 + \cdots x_n$$

$$\begin{aligned} x(t) = \Big\{ & -4.18705e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2}t\right) + 1.36603e^{-\frac{3t}{2}} \sin\left(\frac{\sqrt{3}}{2}t\right) + 0.17e^{-\frac{5t}{2}} \sin\left(\frac{\sqrt{3}}{2}t\right) - 0.622\, e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) \\ & + 0.28\, e^{-\frac{3t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) - 0.08897e^{-\frac{3t}{2}} \sin\left(\frac{3\sqrt{3}}{2}t\right) + 0.461538\, e^{-\frac{3t}{2}} \cos\left(\frac{3\sqrt{3}}{2}t\right) \\ & - 0.26\, e^{-\frac{5t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) + 0.009e^{-\frac{5t}{2}} \sin\left(\frac{3\sqrt{3}}{2}t\right) + 0.06\, e^{-\frac{5t}{2}} \cos\left(\frac{3\sqrt{3}}{2}t\right) - 0.2e^{-\frac{5t}{2}} \sin\left(\frac{9}{2\sqrt{7}}t\right) \\ & + 0.06e^{-\frac{5t}{2}} \cos\left(\frac{9}{2\sqrt{7}}t\right) - 0.004e^{-\frac{5t}{2}} \sin\left(\frac{5\sqrt{3}}{2}t\right) - 0.003\, e^{-\frac{5t}{2}} \cos\left(\frac{5\sqrt{3}}{2}t\right) + \cdots \Big\} \end{aligned}$$

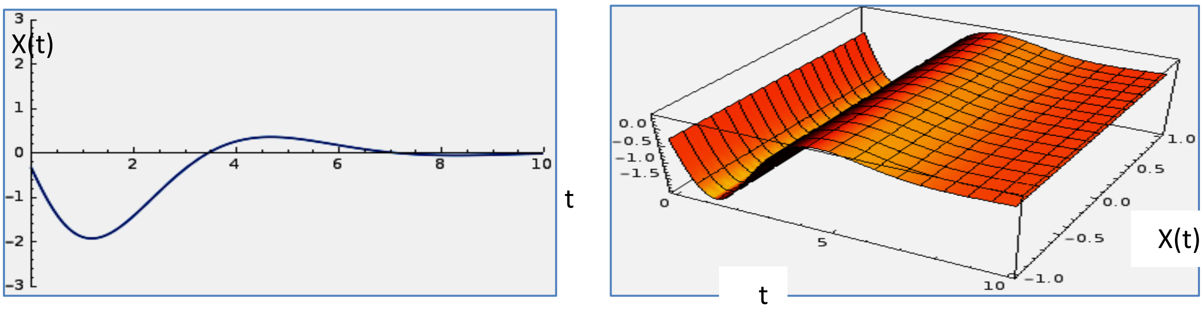

Fig (4): the general approximation solution of Duffing equation.

Notice present the damped of displacement x of a spring with time t, the system returns to equilibrium quickly without oscillating.

CONCLUSIONS :

In this work, the powerful Laplace Adomain Decomposition Method, and its modification were employed for analytical treatment of nonlinear partial differential equations. The unforced Duffing oscillator is considered to show that the modified Adomain Decomposition Method more convenient. And use Walfram Alpha program to plot the solutions.

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